# Chapter 2

**2-1** From Tables A-20, A-21, A-22, and A-24c,

(**a**) UNS G10200 HR: $S_{ut}$ = 380 (55) MPa (kpsi), $S_{yt}$ = 210 (30) Mpa (kpsi) *Ans.*

(**b**) SAE 1050 CD: $S_{ut}$ = 690 (100) MPa (kpsi), $S_{yt}$ = 580 (84) Mpa (kpsi) *Ans.*

(**c**) AISI 1141 Q&T at 540°C (1000°F): $S_{ut}$ = 896 (130) MPa (kpsi), $S_{yt}$ = 765 (111) Mpa (kpsi) *Ans.*

(**d**) 2024-T4: $S_{ut}$ = 446 (64.8) MPa (kpsi), $S_{yt}$ = 296 (43.0) Mpa (kpsi) *Ans.*

(**e**) Ti-6Al-4V annealed: $S_{ut}$ = 900 (130) MPa (kpsi), $S_{yt}$ = 830 (120) Mpa (kpsi) *Ans.*

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**2-2** (**a**) Maximize yield strength: Q&T at 425°C (800°F) *Ans.*

(**b**)Maximize elongation: Q&T at 650°C (1200°F) *Ans.*

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**2-3** Conversion of kN/m$^3$ to kg/ m$^3$ multiply by 1(10$^3$) / 9.81 = 102

AISI 1018 CD steel: Tables A-20 and A-5

$$\frac{S_y}{\rho} = \frac{370\left(10^3\right)}{76.5\left(102\right)} = 47.4 \text{ kN}\cdot\text{m/kg} \qquad \textit{Ans.}$$

2011-T6 aluminum: Tables A-22 and A-5

$$\frac{S_y}{\rho} = \frac{169\left(10^3\right)}{26.6\left(102\right)} = 62.3 \text{ kN}\cdot\text{m/kg} \qquad \textit{Ans.}$$

Ti-6Al-4V titanium: Tables A-24c and A-5

$$\frac{S_y}{\rho} = \frac{830\left(10^3\right)}{43.4\left(102\right)} = 187 \text{ kN}\cdot\text{m/kg} \qquad \textit{Ans.}$$

ASTM No. 40 cast iron: Tables A-24a and A-5.Does not have a yield strength. Using the ultimate strength in tension

$$\frac{S_{ut}}{\rho} = \frac{42.5\left(6.89\right)\left(10^3\right)}{70.6\left(102\right)} = 40.7 \text{ kN}\cdot\text{m/kg} \qquad \textit{Ans}$$

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**2-4**

AISI 1018 CD steel: Table A-5

$$\frac{E}{\gamma} = \frac{30.0\left(10^6\right)}{0.282} = 106\left(10^6\right) \text{ in} \qquad \textit{Ans.}$$

2011-T6 aluminum: Table A-5

$$\frac{E}{\gamma} = \frac{10.4\left(10^6\right)}{0.098} = 106\left(10^6\right) \text{ in} \qquad \textit{Ans.}$$

Ti-6Al-6V titanium: Table A-5

$$\frac{E}{\gamma}=\frac{16.5\left(10^{6}\right)}{0.160}=103\left(10^{6}\right)\ \text{in}\quad \textit{Ans.}$$

No. 40 cast iron: Table A-5

$$\frac{E}{\gamma}=\frac{14.5\left(10^{6}\right)}{0.260}=55.8\left(10^{6}\right)\ \text{in}\quad \textit{Ans.}$$

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**2-5**

$$2G(1+\nu)=E \quad \Rightarrow \quad \nu=\frac{E-2G}{2G}$$

From Table A-5

$$\text{Steel:}\quad \nu=\frac{30.0-2(11.5)}{2(11.5)}=0.304 \quad \textit{Ans.}$$

$$\text{Aluminum:}\quad \nu=\frac{10.4-2(3.90)}{2(3.90)}=0.333 \quad \textit{Ans.}$$

$$\text{Beryllium copper:}\quad \nu=\frac{18.0-2(7.0)}{2(7.0)}=0.286 \quad \textit{Ans.}$$

$$\text{Gray cast iron:}\quad \nu=\frac{14.5-2(6.0)}{2(6.0)}=0.208 \quad \textit{Ans.}$$

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**2-6** (**a**) $A_0 = \pi(0.503)^2/4$, $\sigma = P_i / A_0$

For data in elastic range, $\epsilon = \Delta l / l_0 = \Delta l / 2$

For data in plastic range, $\epsilon=\dfrac{\Delta l}{l_0}=\dfrac{l-l_0}{l_0}=\dfrac{l}{l_0}-1=\dfrac{A_0}{A}-1$

On the next two pages, the data and plots are presented. Figure (*a*) shows the linear part of the curve from data points 1-7. Figure (*b*) shows data points 1-12. Figure (*c*) shows the complete range. **Note**: The exact value of $A_0$ is used without rounding off.

(**b**) From Fig. (*a*) the slope of the line from a linear regression is $E$ = 30.5 Mpsi *Ans.*

From Fig. (*b*) the equation for the dotted offset line is found to be

$$\sigma = 30.5(10^6)\epsilon - 61\ 000 \qquad (1)$$

The equation for the line between data points 8 and 9 is

$$\sigma = 7.60(10^5)\epsilon + 42\ 900 \qquad (2)$$

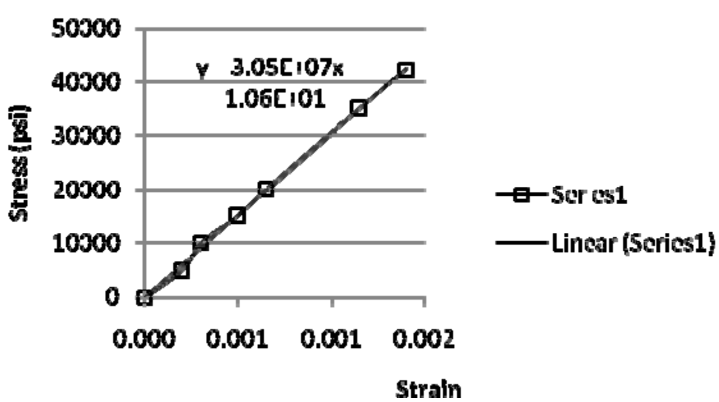
Solving Eqs. (1) and (2) simultaneously yields $\sigma$ = 45.6 kpsi which is the 0.2 percent offset yield strength. Thus, $S_y$ = 45.6 kpsi    *Ans.*

The ultimate strength from Figure (*c*) is $S_u$ = 85.6 kpsi    *Ans.*

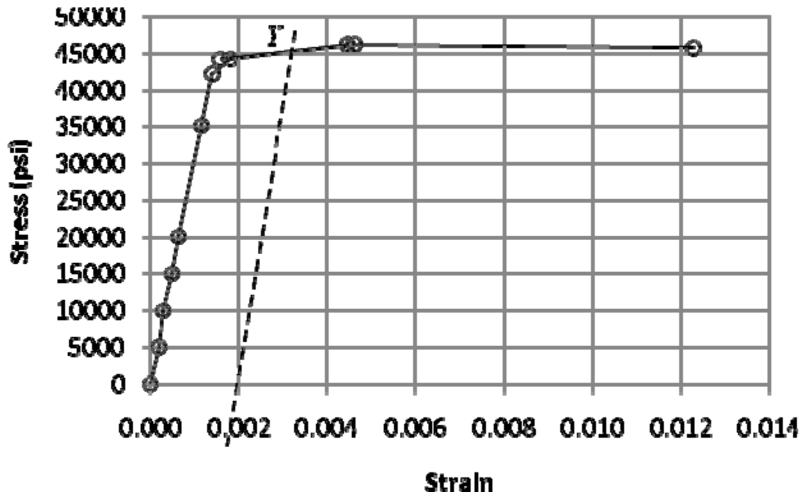
The reduction in area is given by Eq. (2-12) is

$$R = \frac{A_0 - A_f}{A_0}(100) = \frac{0.1987 - 0.1077}{0.1987}(100) = 45.8\ \% \qquad \textit{Ans.}$$

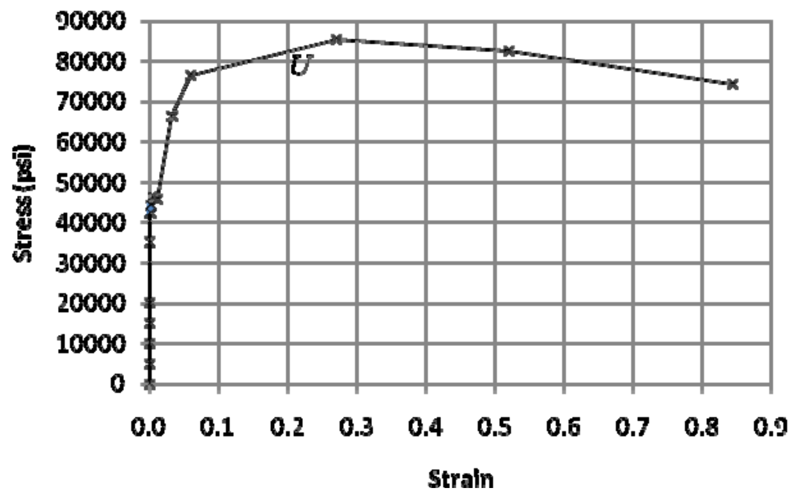
| Data Point | $P_i$ | $\Delta l$, $A_i$ | $\epsilon$ | $\sigma$ |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 2 | 1000 | 0.0004 | 0.00020 | 5032 |
| 3 | 2000 | 0.0006 | 0.00030 | 10065 |
| 4 | 3000 | 0.001 | 0.00050 | 15097 |
| 5 | 4000 | 0.0013 | 0.00065 | 20130 |
| 6 | 7000 | 0.0023 | 0.00115 | 35227 |
| 7 | 8400 | 0.0028 | 0.00140 | 42272 |
| 8 | 8800 | 0.0036 | 0.00180 | 44285 |
| 9 | 9200 | 0.0089 | 0.00445 | 46298 |
| 10 | 8800 | 0.1984 | 0.00158 | 44285 |
| 11 | 9200 | 0.1978 | 0.00461 | 46298 |
| 12 | 9100 | 0.1963 | 0.01229 | 45795 |
| 13 | 13200 | 0.1924 | 0.03281 | 66428 |
| 14 | 15200 | 0.1875 | 0.05980 | 76492 |
| 15 | 17000 | 0.1563 | 0.27136 | 85551 |
| 16 | 16400 | 0.1307 | 0.52037 | 82531 |
| 17 | 14800 | 0.1077 | 0.84506 | 74479 |




(*a*) Linear range




($b$) Offset yield




($c$) Complete range

(**c**) The material is ductile since there is a large amount of deformation beyond yield.

(**d**) The closest material to the values of $S_y$, $S_{ut}$, and $R$ is SAE 1045 HR with $S_y$ = 45 kpsi, $S_{ut}$ = 82 kpsi, and $R$ = 40 %. *Ans.*

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**2-7** To plot $\sigma_{\text{true}}$ vs.$\varepsilon$, the following equations are applied to the data.

$$\sigma_{\text{true}} = \frac{P}{A}$$

Eq. (2-4)

$$\varepsilon = \ln \frac{l}{l_0} \quad \text{for } 0 \le \Delta l \le 0.0028 \text{ in}$$

$$\varepsilon = \ln \frac{A_0}{A} \quad \text{for } \Delta l > 0.0028 \text{ in}$$

where $A_0 = \dfrac{\pi (0.503)^2}{4} = 0.1987 \text{ in}^2$

The results are summarized in the table below and plotted on the next page. The last 5 points of data are used to plot log $\sigma$ vs log $\varepsilon$

The curve fit gives $m = 0.2306$

$$\log \sigma_0 = 5.1852 \Rightarrow \sigma_0 = 153.2 \text{ kpsi} \quad \textit{Ans.}$$

For 20% cold work, Eq. (2-14) and Eq. (2-17) give,

$$A = A_0 (1 - W) = 0.1987 (1 - 0.2) = 0.1590 \text{ in}^2$$

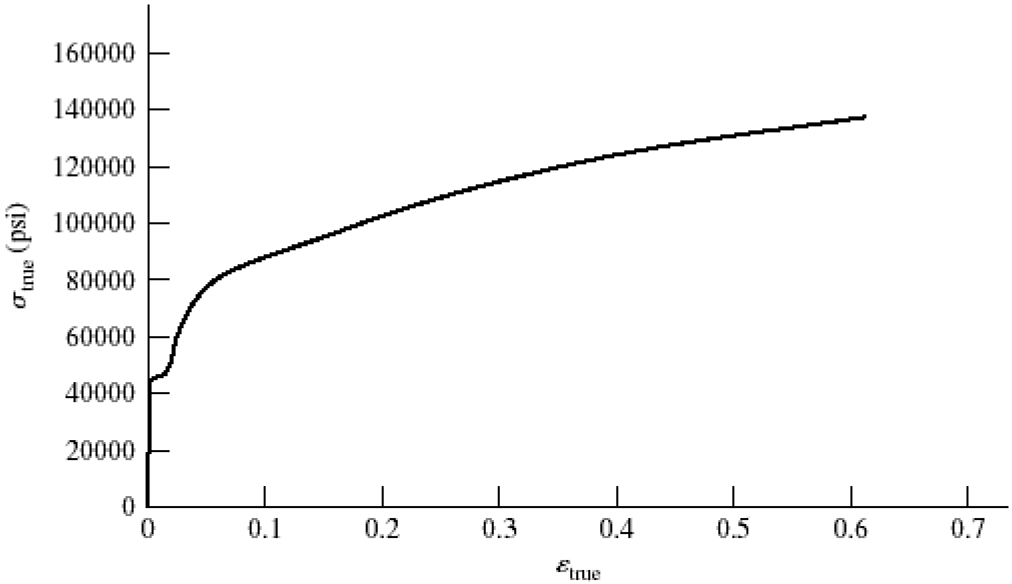
$$\varepsilon = \ln \frac{A_0}{A} = \ln \frac{0.1987}{0.1590} = 0.2231$$

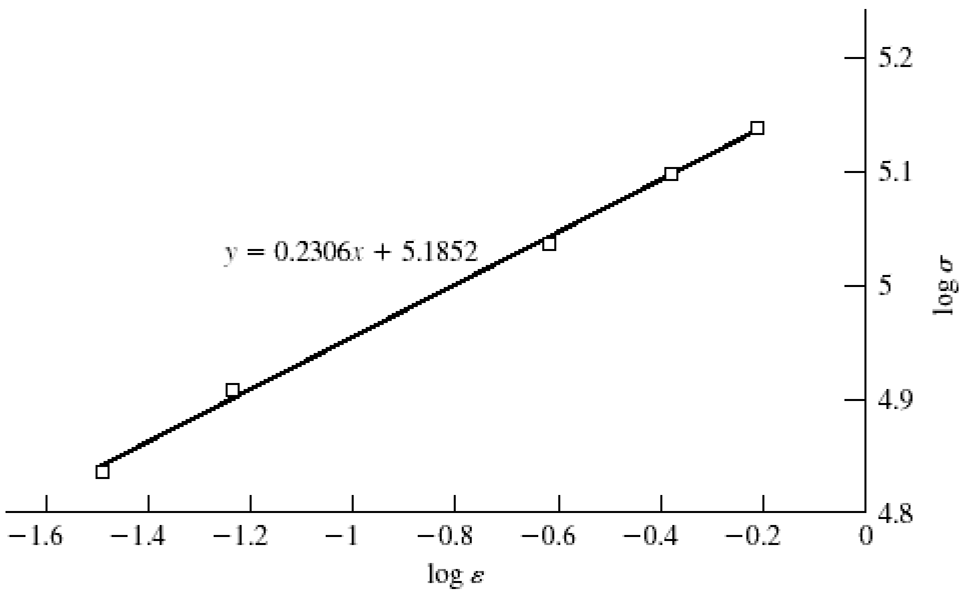
Eq. (2-18): $S_{y'} = \sigma_0 \varepsilon^m = 153.2(0.2231)^{0.2306} = 108.4$ kpsi *Ans.*

Eq. (2-19), with $S_u = 85.6$ from Prob. 2-6,

$$S'_u = \frac{S_u}{1-W} = \frac{85.6}{1-0.2} = 107 \text{ kpsi} \quad \textit{Ans.}$$

| $P$ | $\Delta L$ | $A$ | $\varepsilon$ | $\sigma_{true}$ | log $\varepsilon$ | log $\sigma_{true}$ |
|---|---|---|---|---|---|---|
| 0 | 0 | 0.198 713 | 0 | 0 | | |
| 1000 | 0.0004 | 0.198 713 | 0.000 2 | 5032.388 | -3.699 01 | 3.701 774 |
| 2000 | 0.0006 | 0.198 713 | 0.000 3 | 10 064.78 | -3.522 94 | 4.002 804 |
| 3000 | 0.001 | 0.198 713 | 0.000 5 | 15 097.17 | -3.301 14 | 4.178 895 |
| 4000 | 0.0013 | 0.198 713 | 0.000 65 | 20 129.55 | -3.187 23 | 4.303 834 |
| 7000 | 0.0023 | 0.198 713 | 0.001 149 | 35 226.72 | -2.939 55 | 4.546 872 |
| 8400 | 0.0028 | 0.198 713 | 0.001 399 | 42 272.06 | -2.854 18 | 4.626 053 |
| 8800 | 0.0036 | 0.198 4 | 0.001 575 | 44 354.84 | -2.802 61 | 4.646 941 |
| 9200 | 0.0089 | 0.197 8 | 0.004 604 | 46 511.63 | -2.336 85 | 4.667 562 |
| 9100 | | 0.196 3 | 0.012 216 | 46 357.62 | -1.913 05 | 4.666 121 |
| 13200 | | 0.192 4 | 0.032 284 | 68 607.07 | -1.491 01 | 4.836 369 |
| 15200 | | 0.187 5 | 0.058 082 | 81 066.67 | -1.235 96 | 4.908 842 |
| 17000 | | 0.156 3 | 0.240 083 | 108 765.20 | -0.619 64 | 5.036 49 |
| 16400 | | 0.130 7 | 0.418 956 | 125 478.20 | -0.377 83 | 5.098 568 |
| 14800 | | 0.107 7 | 0.612 511 | 137 418.80 | -0.212 89 | 5.138 046 |

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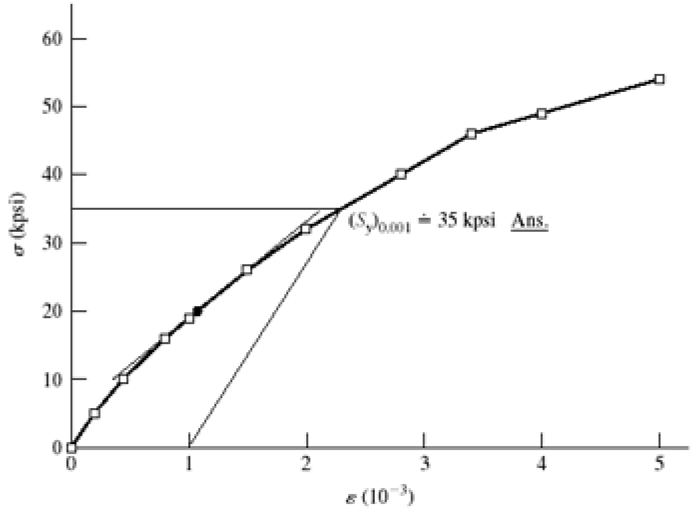
**2-8** Tangent modulus at $\sigma = 0$ is

$$E = \frac{\Delta\sigma}{\Delta\epsilon} \doteq \frac{5000-0}{0.2\left(10^{-3}\right)-0} = 25\left(10^{6}\right) \text{ psi} \qquad \textit{Ans.}$$

At $\sigma = 20$ kpsi

$$E_{20} \doteq \frac{(26-19)(10^3)}{(1.5-1)(10^{-3})} = 14.0(10^6)\ \text{psi} \qquad \textit{Ans.}$$

| $\epsilon$ $(10^{-3})$ | $\sigma$ (kpsi) |
|---|---|
| 0 | 0 |
| 0.20 | 5 |
| 0.44 | 10 |
| 0.80 | 16 |
| 1.0 | 19 |
| 1.5 | 26 |
| 2.0 | 32 |
| 2.8 | 40 |
| 3.4 | 46 |
| 4.0 | 49 |
| 5.0 | 54 |

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**2-9** $W = 0.20$,

(**a**) Before cold working: Annealed AISI 1018 steel. Table A-22, $S_y$ = 32 kpsi, $S_u$ = 49.5 kpsi, $\sigma_0$ = 90.0 kpsi, $m$ = 0.25, $\epsilon_f$ = 1.05

After cold working: Eq. (2-16), $\epsilon_u = m = 0.25$

Eq. (2-14), $\dfrac{A_0}{A_i} = \dfrac{1}{1-W} = \dfrac{1}{1-0.20} = 1.25$

Eq. (2-17), $\varepsilon_i = \ln\dfrac{A_0}{A_i} = \ln 1.25 = 0.223 < \varepsilon_u$

Eq. (2-18), $S_y{}' = \sigma_0 \varepsilon_i^m = 90(0.223)^{0.25} = 61.8$ kpsi *Ans.* 93% increase *Ans.*

Eq. (2-19), $S_u{}' = \dfrac{S_u}{1-W} = \dfrac{49.5}{1-0.20} = 61.9$ kpsi *Ans.* 25% increase *Ans.*

(**b**) Before: $\dfrac{S_u}{S_y} = \dfrac{49.5}{32} = 1.55$ After: $\dfrac{S_u{}'}{S_y{}'} = \dfrac{61.9}{61.8} = 1.00$ *Ans.*

Lost most of its ductility

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**2-10** $W = 0.20$,

(**a**) Before cold working: AISI 1212 HR steel. Table A-22, $S_y$ = 28 kpsi, $S_u$ = 61.5 kpsi, $\sigma_0$ = 110 kpsi, $m$ = 0.24, $\epsilon_f$ = 0.85

After cold working: Eq. (2-16), $\epsilon_u = m = 0.24$

Eq. (2-14), $\frac{A_0}{A_i} = \frac{1}{1-W} = \frac{1}{1-0.20} = 1.25$

Eq. (2-17), $\varepsilon_i = \ln\frac{A_0}{A_i} = \ln 1.25 = 0.223 \ < \varepsilon_u$

Eq. (2-18), $S_y{}' = \sigma_0 \varepsilon_i^m = 110(0.223)^{0.24} = 76.7$ kpsi *Ans.* 174% increase *Ans.*

Eq. (2-19), $S_u{}' = \frac{S_u}{1-W} = \frac{61.5}{1-0.20} = 76.9$ kpsi *Ans.* 25% increase *Ans.*

(**b**) Before: $\frac{S_u}{S_y} = \frac{61.5}{28} = 2.20$ After: $\frac{S_u{}'}{S_y{}'} = \frac{76.9}{76.7} = 1.00$ *Ans.*

Lost most of its ductility

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**2-11** $W = 0.20$,

(**a**) Before cold working: 2024-T4 aluminum alloy. Table A-22, $S_y$ = 43.0 kpsi, $S_u$ = 64.8 kpsi, $\sigma_0$ = 100 kpsi, $m$ = 0.15, $\epsilon_f$ = 0.18

After cold working: Eq. (2-16), $\epsilon_u = m = 0.15$

Eq. (2-14), $\frac{A_0}{A_i} = \frac{1}{1-W} = \frac{1}{1-0.20} = 1.25$

Eq. (2-17), $\varepsilon_i = \ln\frac{A_0}{A_i} = \ln 1.25 = 0.223 \ > \varepsilon_f$ Material fractures. *Ans.*

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**2-12** For $H_B$ = 275, Eq. (2-21), $S_u$ = 3.4(275) = 935 MPa *Ans.*

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**2-13** Gray cast iron, $H_B$ = 200.

Eq. (2-22), $S_u$ = 0.23(200) – 12.5 = 33.5 kpsi *Ans.*

From Table A-24, this is probably ASTM No. 30 Gray cast iron *Ans.*

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**2-14** Eq. (2-21), $0.5H_B = 100 \Rightarrow H_B = 200$ *Ans.*

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**2-15** For the data given, converting $H_B$ to $S_u$ using Eq. (2-21)

| $H_B$ | $S_u$ (kpsi) | $S_u^2$ (kpsi) |
|---|---|---|
| 230 | 115 | 13225 |
| 232 | 116 | 13456 |
| 232 | 116 | 13456 |
| 234 | 117 | 13689 |
| 235 | 117.5 | 13806.25 |
| 235 | 117.5 | 13806.25 |
| 235 | 117.5 | 13806.25 |
| 236 | 118 | 13924 |
| 236 | 118 | 13924 |
| 239 | 119.5 | 14280.25 |
| $\Sigma S_u =$ | 1172 | $\Sigma S_u^2 =$ 137373 |

$$\bar{S}_u = \frac{\sum S_u}{N} = \frac{1172}{10} = 117.2 \doteq 117 \text{ kpsi} \quad \textit{Ans.}$$

Eq. (20-8),

$$s_{S_u} = \sqrt{\frac{\sum_{i=1}^{10} S_u^2 - N\bar{S}_u^2}{N-1}} = \sqrt{\frac{137373 - 10(117.2)^2}{9}} = 1.27 \text{ kpsi} \qquad \textit{Ans.}$$

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**2-16** For the data given, converting $H_B$ to $S_u$ using Eq. (2-22)

| $H_B$ | $S_u$ (kpsi) | $S_u^2$ (kpsi) |
|---|---|---|
| 230 | 40.4 | 1632.16 |
| 232 | 40.86 | 1669.54 |
| 232 | 40.86 | 1669.54 |
| 234 | 41.32 | 1707.342 |
| 235 | 41.55 | 1726.403 |
| 235 | 41.55 | 1726.403 |
| 235 | 41.55 | 1726.403 |
| 236 | 41.78 | 1745.568 |
| 236 | 41.78 | 1745.568 |
| 239 | 42.47 | 1803.701 |
| $\Sigma S_u =$ | 414.12 | $\Sigma S_u^2 =$ 17152.63 |

$$\bar{S}_u = \frac{\sum S_u}{N} = \frac{414.12}{10} = 41.4 \text{ kpsi} \quad \textit{Ans.}$$

Eq. (20-8),

$$s_{S_u} = \sqrt{\frac{\sum_{i=1}^{10} S_u^2 - N\bar{S}_u^2}{N-1}} = \sqrt{\frac{17152.63 - 10(41.4)^2}{9}} = 1.20 \qquad \textit{Ans.}$$

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**2-17** (**a**)

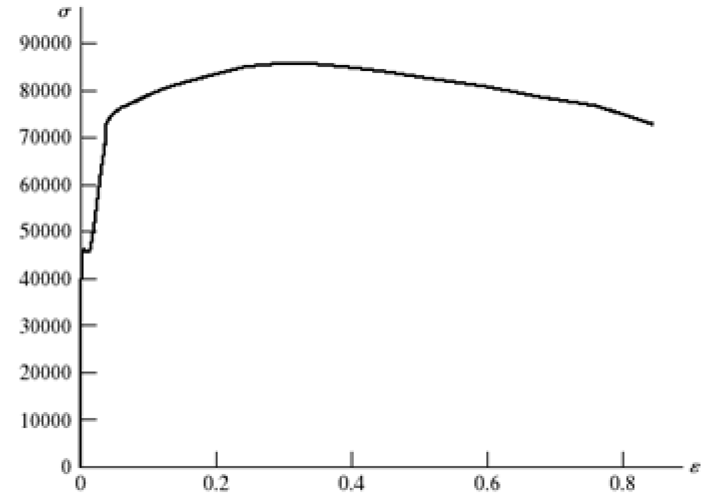
$$u_R \doteq \frac{45.5^2}{2(30)} = 34.5 \text{ in} \cdot \text{lbf / in}^3 \quad \textit{Ans.}$$

(**b**)

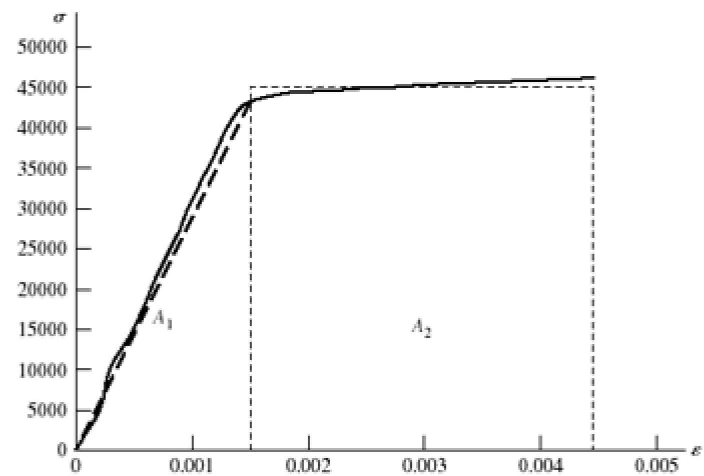
| $P$ | $\Delta L$ | $A$ | $A_0 / A - 1$ | $\epsilon$ | $\sigma = P/A_0$ |
|---|---|---|---|---|---|
| 0 | 0 | | | 0 | 0 |
| 1000 | 0.0004 | | | 0.0002 | 5 032.39 |
| 2000 | 0.0006 | | | 0.0003 | 10 064.78 |
| 3000 | 0.0010 | | | 0.0005 | 15 097.17 |
| 4000 | 0.0013 | | | 0.000 65 | 20 129.55 |
| 7000 | 0.0023 | | | 0.001 15 | 35 226.72 |
| 8400 | 0.0028 | | | 0.0014 | 42 272.06 |
| 8800 | 0.0036 | | | 0.0018 | 44 285.02 |
| 9200 | 0.0089 | | | 0.004 45 | 46 297.97 |
| 9100 | | 0.1963 | 0.012 291 | 0.012 291 | 45 794.73 |
| 13200 | | 0.1924 | 0.032 811 | 0.032 811 | 66 427.53 |
| 15200 | | 0.1875 | 0.059 802 | 0.059 802 | 76 492.30 |
| 17000 | | 0.1563 | 0.271 355 | 0.271 355 | 85 550.60 |
| 16400 | | 0.1307 | 0.520 373 | 0.520 373 | 82 531.17 |
| 14800 | | 0.1077 | 0.845 059 | 0.845 059 | 74 479.35 |

From the figures on the next page,

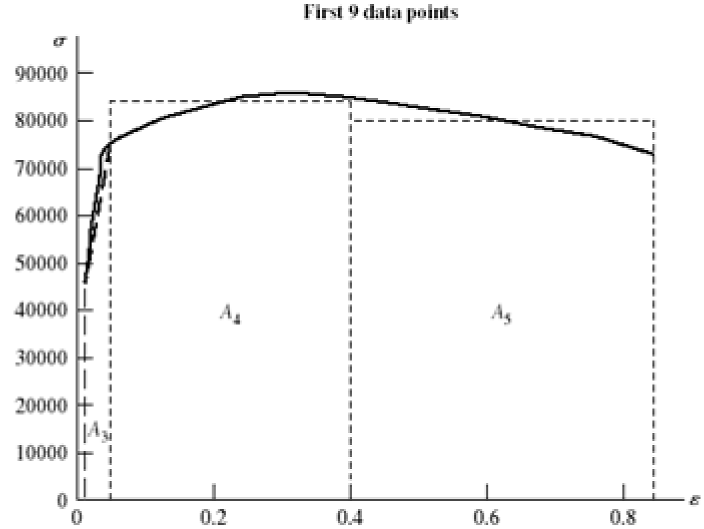
$$u_T \doteq \sum_{i=1}^{5} A_i = \frac{1}{2}(43\ 000)(0.001\ 5) + 45\ 000(0.004\ 45 - 0.001\ 5)$$
$$+\frac{1}{2}(45\ 000 + 76\ 500) + (0.059\ 8 - 0.004\ 45)$$
$$+81\ 000(0.4 - 0.059\ 8) + 80\ 000(0.845 - 0.4)$$
$$\doteq 66.7(10^3) \text{ in} \cdot \text{lbf/in}^3 \quad \textit{Ans.}$$




**All data points**




**First 9 data points**




**Last 6 data points**

**2-18, 2-19** These problems are for student research. No standard solutions are provided.

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**2-20** Appropriate tables: Young's modulus and Density (Table A-5)1020 HR and CD (Table A-20), 1040 and 4140 (Table A-21), Aluminum (Table A-24), Titanium (Table A-24c)

Appropriate equations:

For diameter, $\sigma = \dfrac{F}{A} = \dfrac{F}{(\pi/4)d^2} = S_y \quad \Rightarrow \quad d = \sqrt{\dfrac{4F}{\pi S_y}}$

Weight/length = $\rho A$, Cost/length = \$/in = (\$/lbf) Weight/length,
Deflection/length = $\delta/L = F/(AE)$

With $F$ = 100 kips = 100($10^3$) lbf,

| Material | Young's Modulus | Density | Yield Strength | Cost/lbf | Diameter | Weight/ length | Cost/ length | Deflection/ length |
|---|---|---|---|---|---|---|---|---|
| units | Mpsi | lbf/in^3 | kpsi | \$/lbf | in | lbf/in | \$/in | in/in |
| 1020 HR | 30 | 0.282 | 30 | \$0.27 | 2.060 | 0.9400 | \$0.25 | 1.000E-03 |
| 1020 CD | 30 | 0.282 | 57 | \$0.30 | 1.495 | 0.4947 | \$0.15 | 1.900E-03 |
| 1040 | 30 | 0.282 | 80 | \$0.35 | 1.262 | 0.3525 | \$0.12 | 2.667E-03 |
| 4140 | 30 | 0.282 | 165 | \$0.80 | 0.878 | 0.1709 | \$0.14 | 5.500E-03 |
| Al | 10.4 | 0.098 | 50 | \$1.10 | 1.596 | 0.1960 | \$0.22 | 4.808E-03 |
| Ti | 16.5 | 0.16 | 120 | \$7.00 | 1.030 | 0.1333 | \$0.93 | 7.273E-03 |

The selected materials with minimum values are shaded in the table above. *Ans.*

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**2-21** First, try to find the broad category of material (such as in Table A-5). Visual, magnetic, and scratch tests are fast and inexpensive, so should all be done. Results from these three would favor steel, cast iron, or maybe a less common ferrous material. The expectation would likely be hot-rolled steel. If it is desired to confirm this, either a weight or bending test could be done to check density or modulus of elasticity. The weight test is faster. From the measured weight of 7.95 lbf, the unit weight is determined to be

$$w = \frac{W}{Al} = \frac{7.95\text{ lbf}}{[\pi(1\text{ in})^2/4](36\text{ in})} = 0.281\text{ lbf/in}^3 \doteq 0.28\text{ lbf/in}^3$$

which agrees well with the unit weight of 0.282 lbf/in$^3$ reported in Table A-5 for carbon steel. Nickel steel and stainless steel have similar unit weights, but surface finish and darker coloring do not favor their selection. To select a likely specification from Table

A-20, perform a Brinell hardness test, then use Eq. (2-21) to estimate an ultimate strength of $S_u = 0.5H_B = 0.5(200) = 100 \text{ kpsi}$. Assuming the material is hot-rolled due to the rough surface finish, appropriate choices from Table A-20 would be one of the higher carbon steels, such as hot-rolled AISI 1050, 1060, or 1080. *Ans.*

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**2-22** First, try to find the broad category of material (such as in Table A-5). Visual, magnetic, and scratch tests are fast and inexpensive, so should all be done. Results from these three favor a softer, non-ferrous material like aluminum. If it is desired to confirm this, either a weight or bending test could be done to check density or modulus of elasticity. The weight test is faster. From the measured weight of 2.90 lbf, the unit weight is determined to be

$$w = \frac{W}{Al} = \frac{2.9 \text{ lbf}}{[\pi(1 \text{ in})^2/4](36 \text{ in})} = 0.103 \text{ lbf/in}^3 \doteq 0.10 \text{ lbf/in}^3$$

which agrees reasonably well with the unit weight of 0.098 lbf/in$^3$ reported in Table A-5 for aluminum. No other materials come close to this unit weight, so the material is likely aluminum. *Ans.*

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**2-23** First, try to find the broad category of material (such as in Table A-5). Visual, magnetic, and scratch tests are fast and inexpensive, so should all be done. Results from these three favor a softer, non-ferrous copper-based material such as copper, brass, or bronze. To further distinguish the material, either a weight or bending test could be done to check density or modulus of elasticity. The weight test is faster. From the measured weight of 9 lbf, the unit weight is determined to be

$$w = \frac{W}{Al} = \frac{9.0 \text{ lbf}}{[\pi(1 \text{ in})^2/4](36 \text{ in})} = 0.318 \text{ lbf/in}^3 \doteq 0.32 \text{ lbf/in}^3$$

which agrees reasonably well with the unit weight of 0.322 lbf/in$^3$ reported in Table A-5 for copper. Brass is not far off (0.309 lbf/in$^3$), so the deflection test could be used to gain additional insight. From the measured deflection and utilizing the deflection equation for an end-loaded cantilever beam from Table A-9, Young's modulus is determined to be

$$E = \frac{Fl^3}{3Iy} = \frac{100(24)^3}{3\left(\pi(1)^4/64\right)(17/32)} = 17.7 \text{ Mpsi}$$

which agrees better with the modulus for copper (17.2 Mpsi) than with brass (15.4 Mpsi). The conclusion is that the material is likely copper. *Ans.*

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**2-24 and 2-25** These problems are for student research. No standard solutions are provided.

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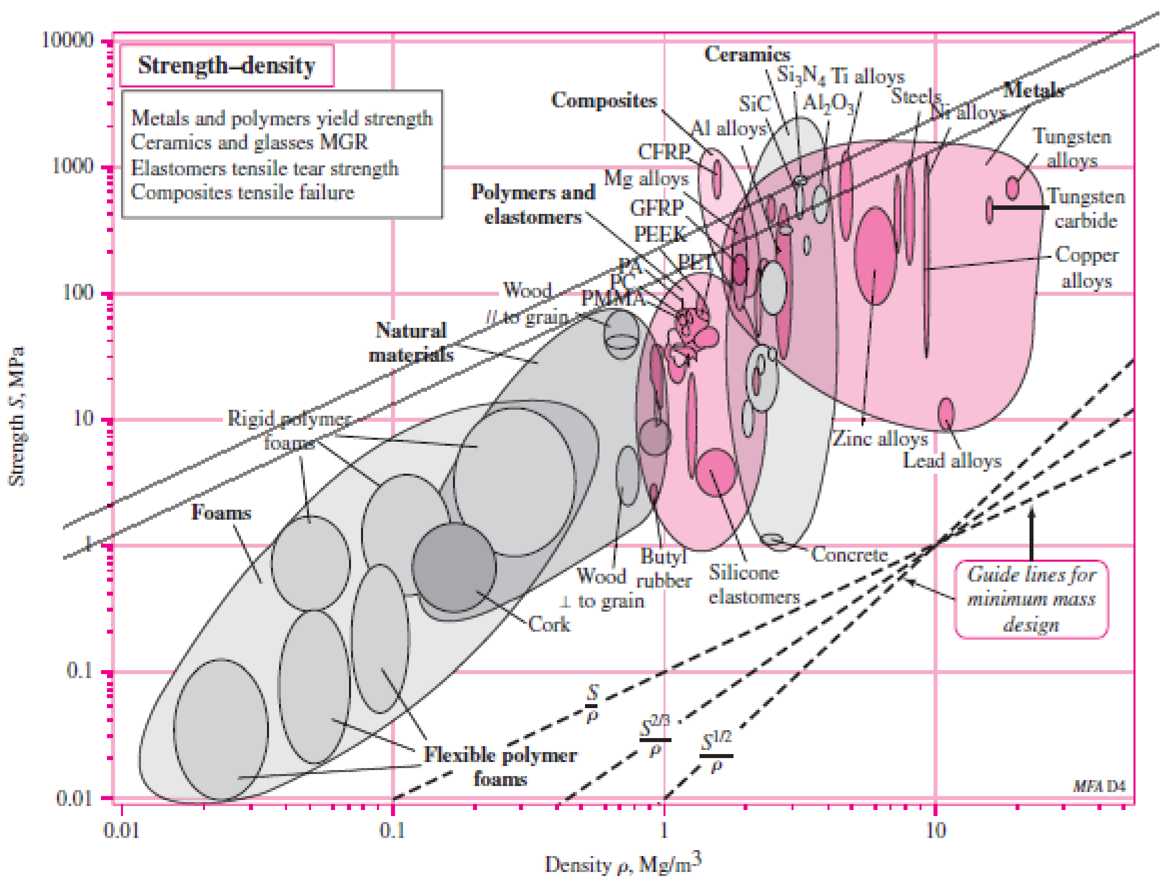
**2-26** For strength, $\sigma = F/A = S \quad \Rightarrow \quad A = F/S$
For mass, $m = Al\rho = (F/S)\, l\rho$

Thus, $f_3(M) = \rho/S$, and maximize $S/\rho \quad (\beta = 1)$

In Fig. (2-19), draw lines parallel to $S/\rho$



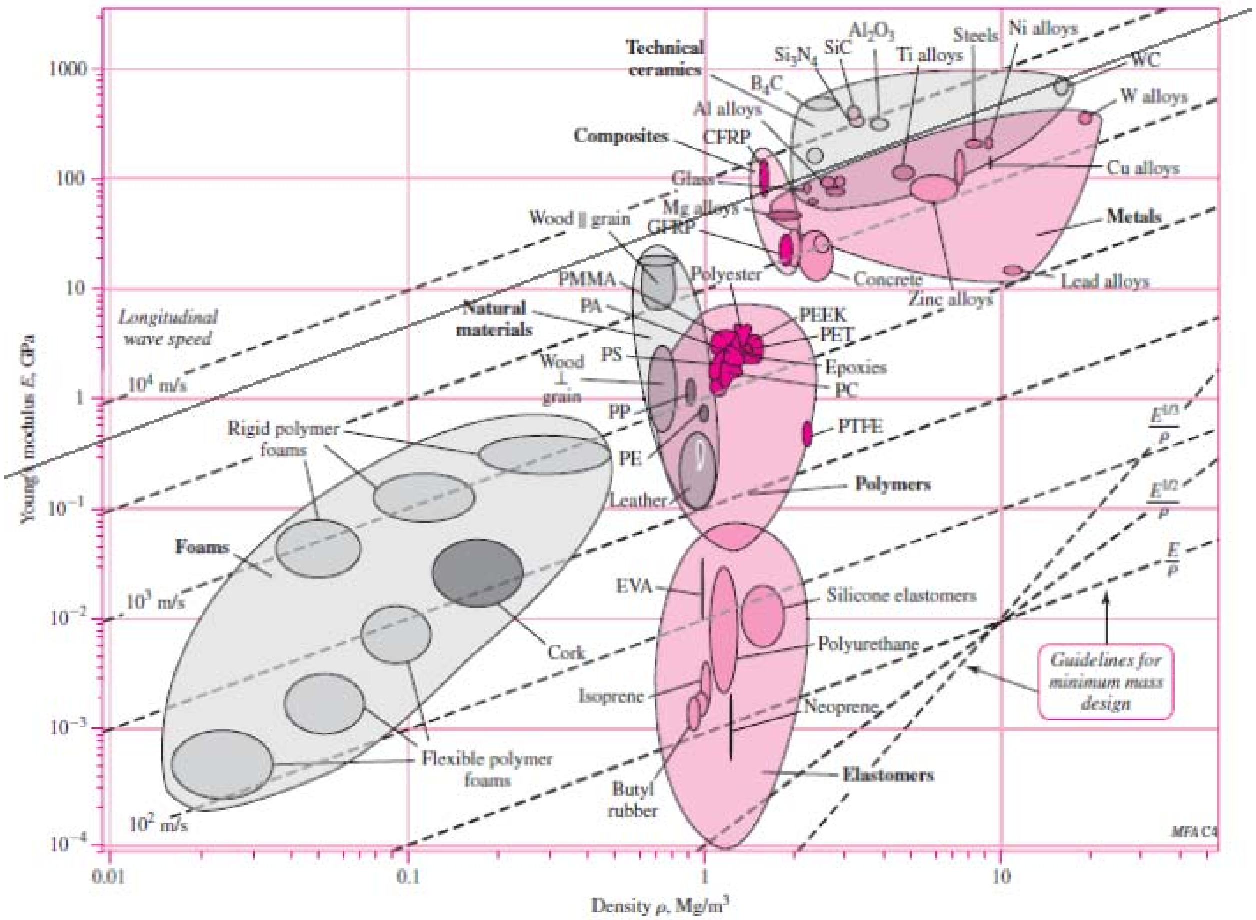

From the list of materials given, both **aluminum alloy** and **high carbon heat treated steel** are good candidates, having greater potential than tungsten carbide or polycarbonate. The higher strength aluminum alloys have a slightly greater potential. Other factors, such as cost or availability, may dictate which to choose. *Ans.*

---

**2-27** For stiffness, $k = AE/l \quad \Rightarrow \quad A = kl/E$
For mass, $m = Al\rho = (kl/E)\, l\rho = kl^2 \rho/E$

Thus, $f_3(M) = \rho/E$, and maximize $E/\rho \quad (\beta = 1)$

In Fig. (2-16), draw lines parallel to $E/\rho$




From the list of materials given, **tungsten carbide** (WC) is best, closely followed by aluminum alloys, and then followed by high carbon heat-treated steel. They are close enough that other factors, like cost or availability, would likely dictate the best choice. Polycarbonate polymer is clearly not a good choice compared to the other candidate materials. *Ans*.

---

**2-28** For strength,

$$\sigma = Fl/Z = S \qquad (1)$$

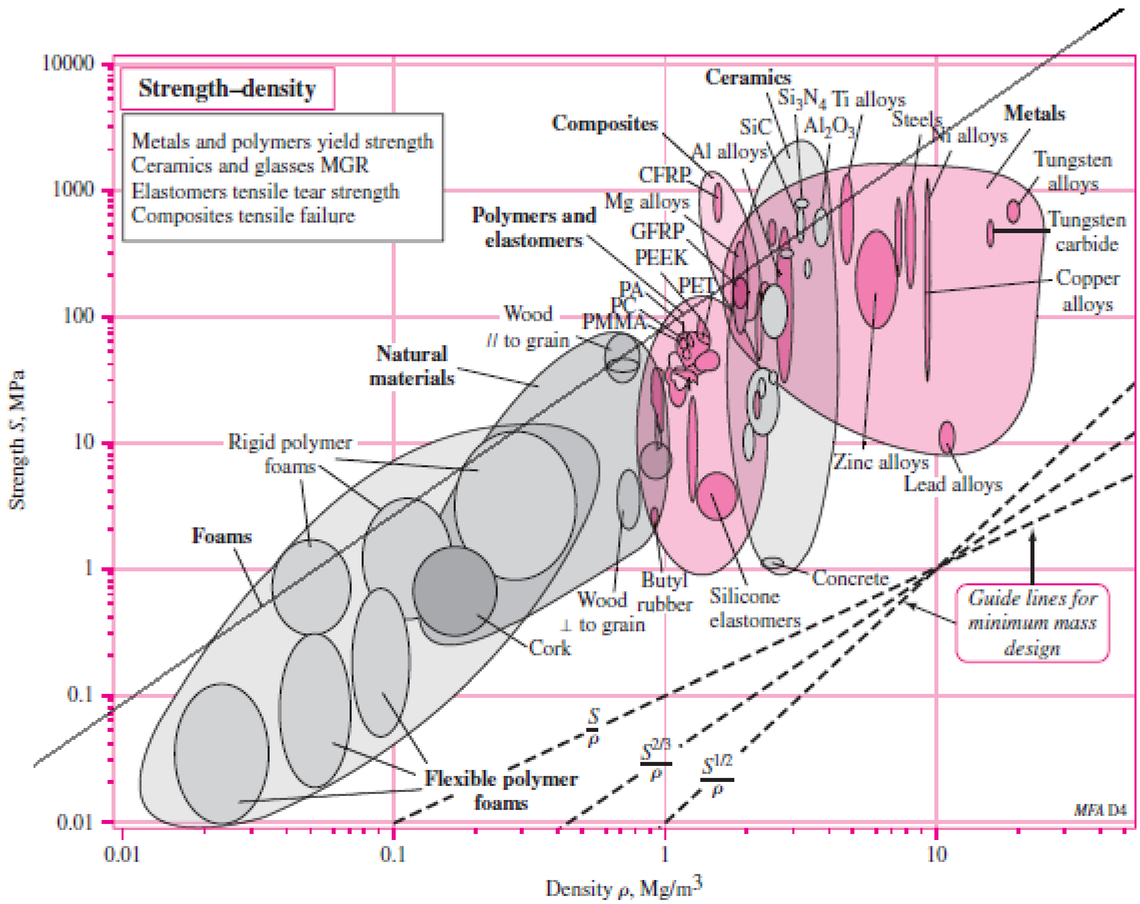
where $Fl$ is the bending moment and $Z$ is the section modulus [see Eq. (3-26*b*), p. 90 ]. The section modulus is strictly a function of the dimensions of the cross section and has the units in$^3$ (ips) or m$^3$ (SI). Thus, for a given cross section, $Z = C\,(A)^{3/2}$, where $C$ is a number. For example, for a circular cross section, $C = \left(4\sqrt{\pi}\right)^{-1}$. Then, for strength, Eq. (1) is

$$\frac{Fl}{CA^{3/2}} = S \quad \Rightarrow \quad A = \left(\frac{Fl}{CS}\right)^{2/3} \qquad (2)$$

For mass, $m = Al\rho = \left(\frac{Fl}{CS}\right)^{2/3} l\rho = \left(\frac{F}{C}\right)^{2/3} l^{5/3}\left(\frac{\rho}{S^{2/3}}\right)$

Thus, $f_3(M) = \rho/S^{2/3}$, and maximize $S^{2/3}/\rho$ $(\beta = 2/3)$

In Fig. (2-19), draw lines parallel to $S^{2/3}/\rho$




From the list of materials given, a higher strength **aluminum alloy** has the greatest potential, followed closely by high carbon heat-treated steel. Tungsten carbide is clearly not a good choice compared to the other candidate materials. *.Ans.*

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**2-29** Eq. (2-26), p. 65, applies to a circular cross section. However, for any cross section *shape* it can be shown that $I = CA^2$, where $C$ is a constant. For example, consider a rectangular section of height $h$ and width $b$, where for a given scaled shape, $h = cb$, where $c$ is a constant. The moment of inertia is $I = bh^3/12$, and the area is $A = bh$. Then $I = h(bh^2)/12 = cb\,(bh^2)/12 = (c/12)(bh)^2 = CA^2$, where $C = c/12$ (a constant).

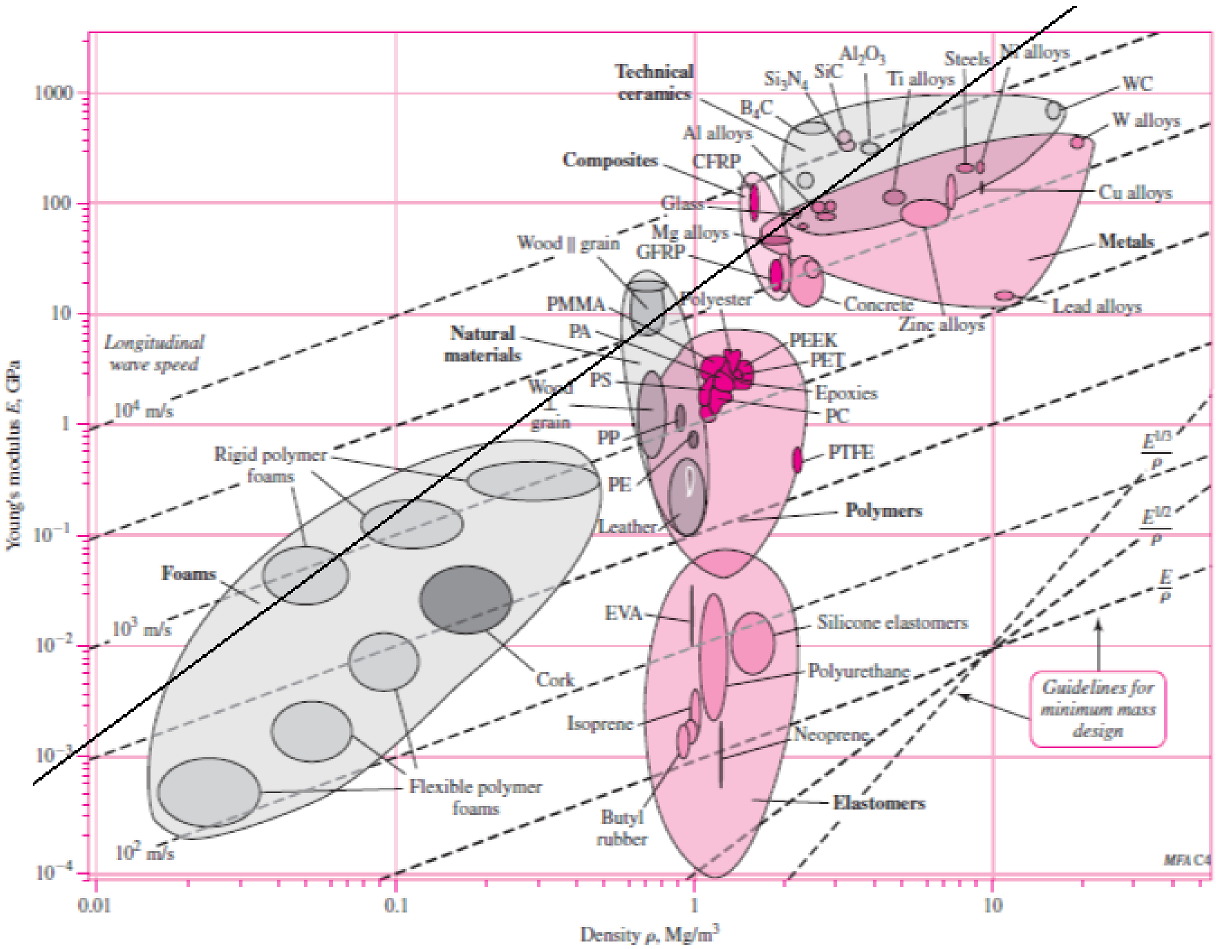
Thus, Eq. (2-27) becomes

$$A = \left(\frac{kl^3}{3CE}\right)^{1/2}$$

and Eq. (2-29) becomes

$$m = Al\rho = \left(\frac{k}{3C}\right)^{1/2} l^{5/2}\left(\frac{\rho}{E^{1/2}}\right)$$

Thus, minimize $f_3(M) = \dfrac{\rho}{E^{1/2}}$, or maximize $M = \dfrac{E^{1/2}}{\rho}$. From Fig. (2-16)




From the list of materials given, **aluminum alloys** are clearly the best followed by steels and tungsten carbide. Polycarbonate polymer is not a good choice compared to the other candidate materials. *Ans.*

---

**2-30** For stiffness, $k = AE/l \;\Rightarrow\; A = kl/E$

For mass, $m = Al\rho = (kl/E)\,l\rho = kl^2\,\rho/E$

So, $f_3(M) = \rho/E$, and maximize $E/\rho$. Thus, $\beta = 1$. *Ans.*

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**2-31** For strength, $\sigma = F/A = S \Rightarrow A = F/S$
For mass, $m = Al\rho = (F/S)\, l\rho$

So, $f_3(M) = \rho/S$, and maximize $S/\rho$. Thus, $\beta = 1$. *Ans.*

---

**2-32** Eq. (2-26), p. 65, applies to a circular cross section. However, for any cross section *shape* it can be shown that $I = CA^2$, where $C$ is a constant. For example, consider a rectangular section of height $h$ and width $b$, where for a given scaled shape, $h = cb$, where $c$ is a constant. The moment of inertia is $I = bh^3/12$, and the area is $A = bh$. Then $I = h(bh^2)/12 = cb\,(bh^2)/12 = (c/12)(bh)^2 = CA^2$, where $C = c/12$.

Thus, Eq. (2-27) becomes

$$A = \left(\frac{kl^3}{3CE}\right)^{1/2}$$

and Eq. (2-29) becomes

$$m = Al\rho = \left(\frac{k}{3C}\right)^{1/2} l^{5/2}\left(\frac{\rho}{E^{1/2}}\right)$$

So, minimize $f_3(M) = \dfrac{\rho}{E^{1/2}}$, or maximize $M = \dfrac{E^{1/2}}{\rho}$. Thus, $\beta = 1/2$. *Ans.*

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**2-33** For strength,

$$\sigma = Fl/Z = S \qquad (1)$$

where $Fl$ is the bending moment and $Z$ is the section modulus [see Eq. (3-26*b*), p. 90 ]. The section modulus is strictly a function of the dimensions of the cross section and has the units in$^3$ (ips) or m$^3$ (SI). Thus, for a given cross section, $Z = C\,(A)^{3/2}$, where $C$ is a number. For example, for a circular cross section, $C = \left(4\sqrt{\pi}\right)^{-1}$. Then, for strength, Eq. (1) is

$$\frac{Fl}{CA^{3/2}} = S \quad \Rightarrow \quad A = \left(\frac{Fl}{CS}\right)^{2/3} \qquad (2)$$

For mass,

$$m = Al\rho = \left(\frac{Fl}{CS}\right)^{2/3} l\rho = \left(\frac{F}{C}\right)^{2/3} l^{5/3}\left(\frac{\rho}{S^{2/3}}\right)$$

So, $f_3(M) = \rho/S^{2/3}$, and maximize $S^{2/3}/\rho$. Thus, $\beta = 2/3$. *Ans.*

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**2-34** <u>For stiffness</u>, $k = AE/l$, or, $A = kl/E$.

Thus, $m = \rho A l = \rho (kl/E)l = kl^2 \rho/E$. Then, $M = E/\rho$ and $\beta = 1$.

From Fig. 2-16, lines parallel to $E/\rho$ for ductile materials include steel, titanium, molybdenum, aluminum alloys, and composites.

For strength, $S = F/A$, or, $A = F/S$.

Thus, $m = \rho A l = \rho F/Sl = Fl \rho/S$. Then, $M = S/\rho$ and $\beta = 1$.

From Fig. 2-19, lines parallel to $S/\rho$ give for ductile materials, steel, aluminum alloys, nickel alloys, titanium, and composites.

Common to both stiffness and strength are steel, titanium, aluminum alloys, and composites. *Ans.*